

Topics for the FSE in Mathematics 2011

Mathematics I

1. Systems of linear algebraic equations. Frobenius theorem. Possibilities of a numerical solution.
2. Investigation of behavior of a function of one real variable – an emphasis on monotonicity, local extrema, convexity, concavity and inflection points using derivatives.
3. Riemann's integrals, sufficient conditions of existence of the integral. Newton-Leibniz formula. Basic methods of calculation (integration by substitution and integration by parts). Geometric applications (area of a two-dimensional region, volume of a rotational body, length of a curve).

Mathematics II

4. Functions of several variables. Partial derivatives and gradients, their geometrical meaning. Tangent plane, an approximate calculation of the value of a function. Directional derivatives.
5. Evaluation of the local extrema of functions of two variables.
6. Double and triple integrals. Evaluation of the integrals using Fubini's theorem. Geometric and physical applications of double and triple integrals (area of a region, volume of a body, mass, moment of inertia, and center of mass).
7. The line integrals of vector functions. Calculation of the integrals using parametrizations of curves.
Evaluation of the work done along a given curve.
8. Surface integrals of vector function. Calculation of the integrals using parametrizations of curves.
Evaluation of the flow of a vector field through a closed surface. Gauss' divergence theorem.

Mathematics III

9. Differential equations of the second order with constant coefficients. The fundamental system of solutions. The general solution of a homogeneous equation.
Particular and general solutions of a non-homogeneous equation.
10. System of linear autonomous differential equations. Calculation of the general solution using eigenvalues and eigenvectors of the system. Types of equilibrium points.
11. Power series, the domain of convergence. Taylor series of functions of one variable (exponential, sine, cosine, arctg, and logarithm functions). Expansion of rational functions in power series using the geometric series.

Numerical mathematics

12. The principle of iterative methods for solution of systems of linear algebraic equations.
Jacobi and Gauss-Seidel iteration method.
Matrix notation and the coordinate notation. Conditions of convergence.
13. Numerical solution of the Cauchy's problem for ordinary differential equation in normal form.
One-step methods of the first and second order.
Principle of numerical solution for systems of ordinary differential equations.
14. Solution of the Dirichlet problem for the Poisson's equation using the finite-difference method.
Construction of a system of finite-difference equations.
Discretization errors, the convergence of the method.